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Formulating Mathematica pseudocodes of block-Milne's device for accomplishing third-order ODEs

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ABSTRACT

Formulating Mathematica pseudocodes for carrying out third-order ordinary differential equations (ODEs) is of essence necessary for proficient computation. This research paper is prepared to formulate Mathematica Pseudocodes block Milne's device (FMPBMD) for accomplishing third-order ODEs. The coming together of Mathematica pseudocodes and proficient computing using block Milne's device will bring about ease in ciphering, proficiency, acceleration and better accuracy. Side by side estimation and extrapolation is considered with successive function approximation gives rise to FMPBMD. This FMPBMD turns out to bring about the star local truncation error thereby finding the degree of the scheme. FMPBMD will be implemented on some numerical examples to corroborate the superiority over other block methods established by employing fixed step size and handled computation.

1. Introduction

The act of unfolding block-predictor-block-corrector methods is all-important to the developmental process of block Milne's device. Especially, looking for closed solution to ODEs. This research paper is proposed in the direction of formulating Mathematica pseudocodes of block Milne's device for looping third-order ODEs (Dormand, 1996; Oghonyon et al., 2015) i.e.:

$$u''' = z(v,u,u',u''), u(b) = b_0, u'(b) = b_1, u''(b) = b_2$$
$$\text{for } c \le v \le d \text{ and } z{:}\times \Re^{\lambda} \to \Re^{\lambda} \quad (1)$$

The approximate resolution to Eq. 1 can be represented broadly as

$$\sum_{n=1}^{m} \varphi_j u_{j+n-1} = h^3 \sum_{n=1}^{m} \theta_j z_{j+n-1} \quad (2)$$

where h is the length measure, $\varphi_m = 1$, $\varphi_n \quad n = 0,1,\ldots,m$, θ_m are specified unknown-quantity with distinctly defined system of degree j (Anake et al., 2012; 2013; Oghonyon et al., 2015; 2016).

Consider for granted that it is tolerable to a justifiable state and meets a planetal assumptions for $\mathcal{L} \ge 0 \ni$

$$|z(v,u) - z^*(v,\bar{\bar{u}})| \le L|u - \bar{\bar{u}}|, \quad \forall u, \bar{\bar{u}} \in \Re,$$

underneath this presumptions, Eq. 1 checks out the planetal and oneness outlined, besides meets the requirements of the Weierstrass theorem (Jain and Iyengar, 2005; Faires and Burden, 2012; Oghonyon et al., 2015).

Writers hinted that the step-down of Eq. 1 to systems of ODEs generates some less favorable consequences. This unfavorable consequence involves some serious setback. This setback includes waste of manpower, difficulty in writing/ implementing programming codes and time consumption. Scholars have developed direct and special methods for solving equation Eq. 1. These path ways constitute block predictor and block corrector method, block implicit method, block hybrid method and backward differentiation method (Anake et al., 2012; 2013; Mohammed and Adeniyi, 2014; Kuboye and Omar, 2015; Olabode, 2009; 2013; Olabode and Yusuph, 2009; Omar and Sulaiman, 2004). Yet, sources have indicated block predictor-corrector method of Adams typecast for working non-stiff ODEs (Dormand, 1996; Awoyemi, 2003; Oghonyon et al., 2015; 2016). Others look at backward differentiation formula (BDF) differently addressed by Gear (1971) for working-out stiff ODEs. Entirely, this research work is put forward to overcome the designs of fixed step-size variation, unable to define converging standards, curb error, exclude BDF which handles stiff ODEs (Majid and Suleiman, 2007; 2008; Langkah et al., 2012;

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Mehrkanoon et al., 2010; Mehrkanoon, 2011; Rauf et al., 2015).

Formulating Mathematica pseudocodes of block Milne's device for accomplishing third-order ODEs is the principal destination of this research study. These path ways of accomplishing Mathematica pseudocodes are built up to give immediate output, skillful and niftier accuracy. But then, block Milne's device is formulated to better converging standards, vary-step-size and curb errors (Dormand, 1996; Faires and Burden, 2012; Lambert, 1991; Oghonyon et al., 2015; 2016).

Definition: Consider x-block, y-point-method and assume x indicates the block-size and value magnitude h, then block-magnitude in time period is yh. Let $w = 0,1,2,\dots$ depict the block measure and $c = wy$, while x-block, y-point-method is the future superior-general figure:

$$U_\mu = \sum_{s=1}^{b} A_s U_{\mu-s} + h\sum_{s=0}^{b} B_s Z_{\mu-s} \tag{3}$$

where

$$U_u = [u_{n+1}, \dots, u_{n+1}]^{\wedge\omega}$$
$$Z_u = [z_{n+1}, \dots, z_{n+1}]^{\wedge\omega}.$$

A_S and B_S are y×y constant-coefficients of arrangement of expressions expressed by rows and columns (Ibrahim et al., 2007).

In addition, for the concise explanation (definition) stated before, block way defines the mathematical gains for real-life coatings and vaulted output is simultaneously generated at more-point. Thus, the amounts of valuates trust on the development of the block method. Employing this approach will supply faster and more improved outputs to the given application which can be calculated to furnish the sought-after truth (Majid and Suleiman, 2007; 2008).

The organization of this research work is as follows: in section 2, Mathematica pseudocodes of block Milne's device is presented; in section 3, Mathematica pseudocodes for accomplishing block Milnes' device is addressed; in section 4, conclusion as seen in Akinfenwa et al. (2013) and Oghonyon et al. (2016) is discussed.

2. Materials and methods

This section is dedicated to formulate pseudocodes of block Milne's device. Block Milne's device is an accumulation of the 5-step-explicit method and 4-step-implicit method respectively. This accumulation is presented as

$$u(v) = \sum_{r=0}^{n} \alpha_r u_{k-r} + h^3 \sum_{r=0}^{n} \beta_r z_{k-r}, \tag{4}$$
$$u(v) = \sum_{r=0}^{n} \alpha_r u_{k-r} + h^3 \sum_{r=1}^{n} \boldsymbol{\beta}_r^* z_{k+r}. \tag{5}$$

Putting together Eq. 4 and Eq. 5 will yield the block Milne's device, where β_r, $r = 0,1,2,3$. Referring to u_{n+r} as the approximate of the exact, results in $u(v_{n+r})$ i.e. $u(v_{n+r}, u_{n+r}) \approx u_{n+r}$ and $z(v_{n+r}, u_{n+r}) \approx z_{n+r}$owning $r = 0,1,2,3$. To realize Eq. 4 and Eq. 5, the power-series approximate is extrapolated and differentiated side-by side about chosen-intervals leading organized system to the linear equation i.e. $Au = v$.

$$u(v) = \sum_{n=0}^{r} a_r \left(\frac{x-x_r}{h}\right)^n. \tag{6}$$

Eq. 6 is converted from ordinary language into code to reproduce the Mathematica pseudocodes as

$$u[v\_] = e[0] + e[1]\frac{(v-v[n])}{h} + e[2]\frac{(v-v[n])^2}{h^2} + e[3]\frac{(v-v[n])^3}{h^3} + e[4]\frac{(v-v[n])^4}{h^4} + e[5]\frac{(v-v[n])^5}{h^5} + +e[6]\frac{(v-v[n])^6}{h^5} + e[7]\frac{(v-v[n])^7}{h^7} + e[8]\frac{(v-v[n])^8}{h^8}, \tag{7}$$

where $e[0], e[1], e[2], e[3],\ e[4], e[5], e[6]$ and $e[7]$ will be considered as unknown-parameters demanded to be checked in specified manner. Pre-suppose that the pre-condition of Eq. 6 aligns with the exact-result at some-selected time-interval $v_n, v_{n-r}$to get the estimate of

$$u(v_n) \approx u, \quad u(v_{n-r}) \approx u_{n-r}. \tag{8}$$

Predicting Eq. 7 matches Eq. 1 at the some-selected-points $v_{n+r}, r = 0,1,2,3$ to develop the next approximates as

$$u'''(v_{n+r}) \approx z_{n+r},\ r = 0,1,2,3. \tag{9}$$

Coming together of the forecasts of Eq. 8 and Eq. 9 will translate into the eight-fold-systems of equation which brings out $Au = x$. Working-out $Au = x$ will result to block Milne's device of the block-predictor-corrector method constituted as the Mathematica pseudocodes

$$matrixa = \left\{ \begin{array}{c} \{1,0,0,0,0,0,0,0\}, \\ \{1,-1,1,-1,1,-1,1,-1\}, \\ \{1,-2,4,-8,16,-32,64,-128\}, \\ \{0,0,0,6,0,0,0,0\}, \\ \{0,0,0,6,-24,60,-120,210\}, \\ \{0,0,0,6,-48,240,-960,3360\}, \\ \{0,0,0,6,-72,540,-3240,17010\}, \\ \{0,0,0,6,-96,960,-7680,53760\} \end{array} \right\}$$

$x = \{u[n], u[n-1], u[n-2], z[n], z[n-1], z[n-2], z[n-3], z[n-4]\}$;
$$\{c, f, g, i, l, m, o, t\} = Inverse[matrixa].x \tag{10}$$

$$matrixa = \left\{ \begin{array}{c} \{1,0,0,0,0,0,0,0\}, \\ \{1,-1,1,-1,1,-1,1,-1\}, \\ \{1,-2,4,-8,16,-32,64,-128\}, \\ \{0,0,0,6,-24,60,-120,210\}, \\ \{0,0,0,6,-72,540,-3240,17010\}, \\ \{0,0,0,6,48,240,960,3360\}, \\ \{0,0,0,6,48,240,960,3360\}, \\ \{0,0,0,6,72,540,3240,17010\} \end{array} \right\};$$

$x = \{u[n], u[n-1], u[n-2], z[n-1], z[n-3], z[n+1], z[n+2], z[n+3]\}$;
$$\{c, f, g, i, l, m, o, t\} = Inverse[matrixa].x \tag{11}$$

to achieve $e[n], n = 0, 1, 2, 3, \dots, 7$ and replacing the measures of $e[n]$ substituted in Eq. 6 to obtain continuous-block Milne's device

$$u[v\_] = \left(1 + \frac{3}{2}\frac{(v-v[n])}{h} + \frac{1}{2}\frac{(v-v[n])^2}{h^2}\right)u[n] + \left(\frac{-2(v-v[n])}{h} - \frac{(v-v[n])^2}{h^2}\right)u[n-1] + \left(\frac{(v-v[n])}{2h} + \frac{(v-v[n])^2}{2h^2}\right)u[n-2] + \left(\frac{307}{5040}\left(\frac{v-v[n]}{h}\right) + \frac{233}{1440}\left(\frac{v-v[n]}{h}\right)^2 + \frac{1}{6}\left(\frac{v-v[n]}{h}\right)^3 + \frac{25}{288}\left(\frac{v-v[n]}{h}\right)^4 + \frac{7}{288}\left(\frac{v-v[n]}{h}\right)^5 + \frac{1}{288}\left(\frac{v-v[n]}{h}\right)^6 + \frac{1}{5040}\left(\frac{v-v[n]}{h}\right)^7\right) z[n]h^3 + \left(\frac{793}{2520}\left(\frac{v-v[n]}{h}\right) + \frac{101}{240}\left(\frac{v-v[n]}{h}\right)^2 - \frac{1}{6}\left(\frac{v-v[n]}{h}\right)^4 - \frac{13}{180}\left(\frac{v-v[n]}{h}\right)^5 - \frac{1}{80}\left(\frac{v-v[n]}{h}\right)^6 - \frac{1}{1260}\left(\frac{v-v[n]}{h}\right)^7\right) z[n-1]h^3 + \left(\frac{-19}{280}\left(\frac{v-v[n]}{h}\right) - \frac{31}{240}\left(\frac{v-v[n]}{h}\right)^2 + \frac{1}{8}\left(\frac{v-v[n]}{h}\right)^4 + \frac{19}{240}\left(\frac{v-v[n]}{h}\right)^5 + \frac{1}{60}\left(\frac{v-v[n]}{h}\right)^6 + \frac{1}{840}\left(\frac{v-v[n]}{h}\right)^7\right) z[n-2]h^3 + \left(\frac{79}{2520}\left(\frac{v-v[n]}{h}\right) + \frac{41}{720}\left(\frac{v-v[n]}{h}\right)^2 - \frac{1}{18}\left(\frac{v-v[n]}{h}\right)^4 - \frac{7}{180}\left(\frac{v-v[n]}{h}\right)^5 - \frac{7}{720}\left(\frac{v-v[n]}{h}\right)^6 - \frac{1}{1260}\left(\frac{v-v[n]}{h}\right)^7\right) f[n-3]h^3 + \left(\frac{-29}{5040}\left(\frac{v-v[n]}{h}\right) - \frac{1}{96}\left(\frac{v-v[n]}{h}\right)^2 + \frac{1}{96}\left(\frac{v-v[n]}{h}\right)^4 + \frac{11}{1440}\left(\frac{v-v[n]}{h}\right)^5 + \frac{1}{480}\left(\frac{v-v[n]}{h}\right)^6 + \frac{1}{5040}\left(\frac{v-v[n]}{h}\right)^7\right) z[n-4]h^3 \quad (12)$$

$$u[v\_] = \left(1 + \frac{3}{2}\frac{(v-v[n])}{h} + \frac{1}{2}\frac{(v-v[n])^2}{h^2}\right)u[n] + \left(\frac{-2(v-v[n])}{h} - \frac{(v-v[n])^2}{h^2}\right)u[n-1] + \left(\frac{(v-v[n])}{2h} + \frac{(v-v[n])^2}{2h^2}\right)u[n-2] + \left(\frac{2801}{10080}\left(\frac{v-v[n]}{h}\right) + \frac{117}{320}\left(\frac{v-v[n]}{h}\right)^2 + \frac{1}{16}\left(\frac{v-v[n]}{h}\right)^3 - \frac{3}{128}\left(\frac{v-v[n]}{h}\right)^4 + \frac{7}{2880}\left(\frac{v-v[n]}{h}\right)^5 + \frac{1}{1920}\left(\frac{v-v[n]}{h}\right)^6 - \frac{1}{10080}\left(\frac{v-v[n]}{h}\right)^7\right) z[n-1]h^3 + \left(\frac{-113}{50400}\left(\frac{v-v[n]}{h}\right) - \frac{97}{14400}\left(\frac{v-v[n]}{h}\right)^2 - \frac{1}{240}\left(\frac{v-v[n]}{h}\right)^3 + \frac{1}{1152}\left(\frac{v-v[n]}{h}\right)^4 + \frac{1}{2880}\left(\frac{v-v[n]}{h}\right)^5 - \frac{1}{5760}\left(\frac{v-v[n]}{h}\right)^6 + \frac{1}{50400}\left(\frac{v-v[n]}{h}\right)^7\right) z[n-3]h^3 + \left(\frac{121}{1120}\left(\frac{v-v[n]}{h}\right) + \frac{251}{960}\left(\frac{v-v[n]}{h}\right)^2 + \frac{3}{16}\left(\frac{v-v[n]}{h}\right)^3 + \frac{3}{128}\left(\frac{v-v[n]}{h}\right)^4 - \frac{11}{960}\left(\frac{v-v[n]}{h}\right)^5 - \frac{1}{1920}\left(\frac{v-v[n]}{h}\right)^6 + \frac{1}{3360}\left(\frac{v-v[n]}{h}\right)^7\right) z[n+1]h^3 + \left(\frac{-101}{1575}\left(\frac{v-v[n]}{h}\right) - \frac{23}{150}\left(\frac{v-v[n]}{h}\right)^2 - \frac{1}{10}\left(\frac{v-v[n]}{h}\right)^3 + \frac{1}{90}\left(\frac{v-v[n]}{h}\right)^5 - \frac{1}{3150}\left(\frac{v-v[n]}{h}\right)^7\right) z[n+2]h^3 + \left(\frac{139}{10080}\left(\frac{v-v[n]}{h}\right) - \frac{19}{576}\left(\frac{v-v[n]}{h}\right)^2 + \frac{1}{48}\left(\frac{v-v[n]}{h}\right)^3 - \frac{1}{1152}\left(\frac{v-v[n]}{h}\right)^4 - \frac{7}{2880}\left(\frac{v-v[n]}{h}\right)^5 + \frac{1}{5760}\left(\frac{v-v[n]}{h}\right)^6 + \frac{1}{10080}\left(\frac{v-v[n]}{h}\right)^7\right) z[n+3]h^3. \quad (13)$$

Appraising Eq. 12 and Eq. 13 at some specific pick-out bound of targets v_{n+r}, $r = 1, 2, 3$ will bring-forth FMPBMD as

$$u[v\_] = e[0]u[n] + e[1]u[n-1] + e[2]u[n-2] + h^3(\beta[0]z[n] + \beta[1]z[n-1] + \beta[2]z[n-2] + \beta[3]z[n-3] + \beta[4]z[n-4]) \quad (14)$$

$$u[v\_] = e[0]u[n] + e[1]u[n-1] + e[2]u[n-2] + h^3(\beta[0]z[n-1] + \beta[1]z[n-3] + \beta[2]z[n+1] + \beta[3]z[n+2] + \beta[4]z[n+3]), \quad (15)$$

where $e[r], r = 0, \dots 4$ and $\beta[r], r = 0,1 \dots 4$ are acknowledged physical-quantity of the FMPBMD (Faires and Burden, 2012).

2.1. Formulating the converging bound FMPBMD

To implement the FMPBMD, 5-step block-explicit method and 4-step block-implicit method are distributed as predictor-corrector pair off holding the like-order. The conflux of Ascher and Petzold (1998), Dormand (1996), Faires and Burden (2012), Lambert (1991), and Oghonyon et al. (2015, 2016) together with the effort of assimilators, it turns more technical to find approximate of star local truncation error of FMPBMD free from estimating derivations of $u(v)$. Presume $\widetilde{p_1} = \overline{\overline{p_2}}$ where $\widetilde{p_1}$and $\overline{\overline{p_2}}$ manifests as the order of block- explicit and block-implicit methods. Straight off, method of order $\widetilde{p_1}$, the 5-step block -explicit method is seen to yield the star local truncation errors:

$$\begin{aligned} \tilde{C}^{[1]}_{\tilde{p}+5} h^{\tilde{p}+5} u^{(\tilde{p}+5)}(v_n) &= u(v_{n+1}) - u^{[q_1]}_{n+1} + O(h^{\tilde{p}+6}), \\ \tilde{C}^{[2]}_{\tilde{p}+5} h^{\tilde{p}+5} u^{(\tilde{p}+5)}(v_n) &= u(v_{n+2}) - u^{[q_2]}_{n+2} + O(h^{\tilde{p}+6}), \\ \tilde{C}^{[3]}_{\tilde{p}+5} h^{\tilde{p}+5} u^{(\tilde{p}+5)}(v_n) &= u(v_{n+3}) - u^{[q_3]}_{n+1} + O(h^{\tilde{p}+6}). \end{aligned} \quad (16)$$

Likewise, the mathematical investigation of 4-step block- implicit method gives the star local truncation errors:

$$\begin{aligned} \bar{C}^{[1]}_{\bar{\bar{p}}+9} h^{\bar{\bar{p}}+5} y^{(\bar{\bar{p}}+5)}(v_n) &= u(v_{n+1}) - u^{[l_1]}_{n+1} + O(h^{\bar{\bar{p}}+6}), \\ \bar{C}^{[2]}_{\bar{\bar{p}}+5} h^{\bar{\bar{p}}+5} y^{(\bar{\bar{p}}+5)}(v_n) &= u(v_{n+2}) - u^{[l_2]}_{n+2} + O(h^{\bar{\bar{p}}+6}), \\ \bar{C}^{[3]}_{\bar{\bar{p}}+5} h^{\bar{\bar{p}}+5} u^{(\bar{\bar{p}}+5)}(v_n) &= u(v_{n+3}) - u^{[l_3]}_{n+3} + O(h^{\bar{\bar{p}}+6}), \end{aligned} \quad (17)$$

where $\tilde{C}^{[1]}_{p+5}, \tilde{C}^{[2]}_{p+5}, \tilde{C}^{[1]}_{p+5}$, $\bar{C}^{[1]}_{\bar{\bar{p}}+5}, \bar{C}^{[2]}_{\bar{\bar{p}}+5}$ and $\bar{C}^{[3]}_{\bar{\bar{p}}+5}$ occur as distinctive physical element irrespective of the varying-step-size h and $u(v)$ is given as analytical resolution of the third-order differential equations gratifying the pre-initial assumption $u(v_n) \approx u_n$.

In moving on, assuming for a smaller-scale measures of h is recognized as follows:

$$u^{(5)}(\tilde{v}_n) \approx u^{(5)}(\bar{v}_n);$$

and as such, generates the converging bounds and accomplishing the FMPBMD banks on the earlier stated presumption.

Valuation of the computational construction of Eq. 16 and Eq. 17 stated earlier, avoiding interference, withdrawing termini of degree $O(h^{p+6})$, it suits the computed star local truncation errors of FMPBMD encountered as

$$\begin{aligned} \bar{C}^{[1]}_{\bar{p}+5} h^{\bar{p}+5} u^{(\bar{p}+5)}(\bar{v}_n) &\approx \frac{\bar{C}^{[1]}_{\bar{p}+5}}{\tilde{C}^{[1]}_{\tilde{p}+5} - \bar{C}^{[1]}_{\bar{p}+5}} \left[u^{[q_1]}_{n+1} - u^{[l_1]}_{n+1}\right] < \varepsilon_1, \\ \bar{C}^{[2]}_{\bar{p}+5} h^{\bar{p}+5} u^{(\bar{p}+5)}(\bar{v}_n) &\approx \frac{\bar{C}^{[2]}_{\bar{p}+9}}{\tilde{C}^{[2]}_{\tilde{p}+5} - \bar{C}^{[2]}_{\bar{p}+5}} \left[u^{[q_2]}_{n+2} - u^{[l_2]}_{n+2}\right] < \varepsilon_2, \\ \bar{C}^{[3]}_{\bar{p}+5} h^{\bar{p}+5} u^{(\bar{p}+5)}(\bar{v}_n) &\approx \frac{\bar{C}^{[3]}_{\bar{p}+9}}{\tilde{C}^{[3]}_{\tilde{p}+5} - \bar{C}^{[3]}_{\bar{p}+5}} \left[u^{[q_3]}_{n+3} - u^{[l_3]}_{n+3}\right] < \varepsilon_3. \end{aligned} \quad (18)$$

Keeping that $u^{[q_1]}_{n+1} \neq u^{[l_1]}_{n+1}$, $u^{[q_2]}_{n+2} \neq u^{[l_2]}_{n+2}$ and $u^{[q_3]}_{n+3} \neq u^{[l_3]}_{n+3}$ were seen as valuates of block-explicit and block-implicit methods got by FMPBMD of order $\bar{\bar{p}}$, while $\bar{C}^{[1]}_{\bar{\bar{p}}+5}h^{\bar{\bar{p}}+5}u^{(\bar{\bar{p}}+5)}(\bar{v}_n)$, $\bar{C}^{[2]}_{\bar{\bar{p}}+5}h^{\bar{\bar{p}}+5}u^{(\bar{\bar{p}}+5)}(\bar{v}_n)$ and $\bar{C}^{[3]}_{\bar{\bar{p}}+5}h^{\bar{\bar{p}}+5}u^{(\bar{\bar{p}}+5)}(\bar{v}_n)$ defines distinctively as star local truncation errors while then ε_1, ε_2 and ε_3 represents boundaries of the converging converging bound.

Even so, star local truncation error of Eq. 18 is exploited to make decision of acceptance or rejection of the successive iteration or re-perform with a refine smaller varying-step-size. This procedure is justly acceptable on test of examination executed by Eq. 18 as seen earlier. For more particulars interested readers can see Ascher and Petzold (1998), Dormand (1996), Faires and Burden (2012), Lambert (1991), and Oghonyon et al. (2015, 2016). Again, the start local truncation errors Eq. 18 is mentioned as the converging bounds of FMPBMD for rectifying convergence.

3. Result and discussion

This section shows the performance of block-Milne's device accomplishing third-order ODEs using formulating Mathematica pseudocodes. The fulfilled computational result issued is got engaging Mathematica 9 Kernel. See FMPBMD ciphers. The language stated in Table 1 is seen underneath:

Problem-Tested: Two problems are tested and accomplished applying FMPBMD on distinctively converging bounds of 0.00000001, 0.000000001, 0.0000000001 and 0.00000000001 (Kuboye and Omar, 2015; Olabode, 2009; 2013; Olabode and Yusuph, 2009; Omar and Sulaiman, 2004).

Tested-Problem 1:
$u'''(v) = -e^v,\quad u(0) = 1,\quad u'(0) = -1,\quad u''(0) = 3.$
Exact-Solution:
$u(x) = 2v^2 - e^v + 2.$
Tested-Problem 2:
$u'''(v) = 3sinv, u(0) = 1,\ u'(0) = 0, u''(0) = -2.$
Exact-Solution:
$u(v) = 3cosv + \frac{v^2}{2} - 2.$

Table 1 and Table 2 show the finished computational results of the tested-problem 1 and 2 applying FMPBMD comparable to existing methods. The nomenclature used in Table 1 and Table 2 are seen infra.

Table 1: Problem 1

Method	Max_{err}	C_{bounds}
ANBM	$7.2263E-8$	10^{-8}
FMPBMD	$6.3496E-8$	10^{-8}
FMPBMD	$6.38009E-8$	
FMPBMD	$6.41019E-8$	
AASBM	$9.73655E-9$	10^{-9}
FMPBMD	$6.07475E-9$	10^{-9}
FMPBMD	$7.42455E-9$	
FMPBMD	$8.77572E-9$	
PR-PIBM	$6.189094E-11$	10^{-11}
FMPBMD	$5.53667E-11$	10^{-11}
FMPBMD	$5.55306E-11$	
FMPBMD	$5.57943E-11$	

Table 2: Problem 2

Method	Max_{err}	C_{bounds}
BMM	$8.35700E-8$	10^{-8}
FMPBMD	$6.01206E-8$	10^{-8}
FMPBMD	$6.02424E-8$	
FMPBMD	$6.03605E-8$	
NS	$8.343294E-10$	10^{-10}
FMPBMD	$6.12059E-10$	10^{-10}
FMPBMD	$6.2424E-10$	
FMPBMD	$6.3654E-10$	

where:
FMPBMD: error in FMPBMD (Formulating Mathematica Pseudocodes of Block Milne's Device for Accomplishing Third-Order Ordinary Differential Equations).
C_{bounds}: converging bounds.
Mth: method used.
Max_{err}: magnitude of the computational maximum errors of FMPBMD.
AASBMO: error in AASBMO (An Accurate Scheme By Block Method for Third Order Ordinary Differential Equations) for tested-problem 1 as cited Olabode (2009).
ANBMS: error in ANBMS (A New Block Method for Special Third-Order ODEs) for tested-problem 1 as seen in Olabode and Yusuph (2009).
BMMDS: error in BMMDS (Block Multistep Method for the Direct Solution of Third-Order of Ordinary Differential Equations) for tested-problem 2 (Olabode, 2013).
NSO: error in NSO (Numerical Solution of Third-Order Ordinary Differential Equations) for tested-problem 2 as seen Kuboye and Omar, 2015.
PR-PIBMH: error in PR-PIBMHO (Parallel R-Point Implicit Block Method for Solving Higher Order Ordinary Differential Equations Directly Using Multistep Collocation Approach) for tested-problem 1 as discoursed (Omar and Sulaiman, 2004).

4. Conclusion

The computational results achieved in Table 1 of problem 1 and Table 2 of problem 2 are truly a force of the converging bounds and varying-step-size. The termini computational result besides prove the functional performance of the FMPBMD to possess a meliorated result than AASBMO, ANBMS, BMMDS, NSO, PR-PIBMHO when in equivalence to kuboye and Omar (2015), Olabode (2009, 2013), Olabode and Yusuph (2009), and Omar and Sulaiman (2004).

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